Weed Technology. 2005. Volume 19:972–978

The Absolute-Log Method of Quantifying Relative Competitive Ability and Niche Differentiation[1]

MICHAEL F. CARPINELLI[2]

Abstract: Plant competition studies designed to quantify interference between species provide valuable information on competitive interactions and on the effects of agronomic practices on those interactions. The effect of each species' density on the growth of itself and on the growth of the other species is quantified in a series of regression models. Traditionally, the models' regression coefficients have been combined in a series of ratios to quantify relative competitive ability and niche differentiation. Coefficients that are negative (positive interference—facilitation, mutualism) or zero (neutral interference or nonsignificant coefficient) do not lend themselves well to ratio-based methodology because of sign cancellation or undefined values, respectively. As a result, ratio-based methodology is limited to using only positive coefficients (negative interference—amensalism, competition). Rather than using ratios, the absolute-log method uses addition and subtraction of coefficients converted to a pseudologarithmic scale, thus allowing for use of coefficients with values that are negative or zero. As a result, the absolute-log method can be used to quantify relative competitive ability and niche differentiation involving all types of interference—negative, positive, and neutral. The absolute-log method includes an optional statistical procedure constructing confidence intervals for the estimates of relative competitive ability and niche differentiation.

Additional index words: Interference, competition, amensalism, facilitation, mutualism, Spitters, reciprocal yield law.

Abbreviations: A-A, absolute-antilog; AND, absolute-antilog function corollary to ND; ARC, absolute-antilog function corollary to RC; CI, confidence interval; LND, absolute-log method corollary to ND; LRC, absolute-log method corollary to RC; ND, niche differentiation value; RC, relative competitive ability value; SE, standard error.

INTRODUCTION

Plant competition studies designed to quantify interference between species provide valuable information to those studying crop–weed interactions. Such studies may also aid revegetation specialists selecting desirable species to compete with a particular weed species or testing the effect of a management action (e.g., fertilization) on the competitive balance between a weed and a desirable species. Several experimental designs attempt to quantify intra- and interspecific interference between plant species. Although they were initially developed to assess the effect of weeds on crops, they can be used to quantify interference among any species. In one such design, the addition series, densities and proportions of different species are systematically varied, and the combined effect on the average weight per plant of each species is quantified.

Analysis of the addition series requires expanding the reciprocal yield law equation to model yield-density responses of two-species systems (Spitters 1983):

$$w_a^{-1} = \beta_{0a} + \beta_{aa}N_a + \beta_{ab}N_b, \qquad [1]$$

where w_a is the mean weight per plant of species *a*, N_a and N_b are the neighbor densities of species *a* and *b*, respectively, and β_{0a}, β_{aa}, and β_{ab} are regression coefficients. The regression coefficients are interpreted as ${\beta_{0a}}^{-1}$ = the yield or weight of an isolated plant of species *a*, β_{aa} is the intraspecific competition coefficient for species *a*, and β_{ab} is the interspecific competition coefficient, or the effect of species *b* on the yield of species *a*. A similar equation can be written for species *b*:

$$w_b^{-1} = \beta_{0b} + \beta_{bb}N_b + \beta_{ba}N_a, \qquad [2]$$

where w_b is the mean weight per plant of species *b*, N_b and N_a are the neighbor densities of species *b* and *a*,

[1] Received for publication January 25, 2005, and in revised form April 23, 2005.

[2] Rangeland Scientist, United States Department of Agriculture, Agricultural Research Service, Burns, OR 97720. Current address: Rangeland Management Specialist, United States Department of Agriculture, Natural Resources Conservation Service, Grants Soil Survey Office, 1041 Mesa Blvd., Suite C, Grants, NM 87020. Corresponding author's E-mail: carpinelli_michael@yahoo.com.

respectively, and β_{0b}, β_{bb}, and β_{ba} are regression coefficients. The regression coefficients are interpreted as β_{0b}^{-1} = the yield or weight of an isolated plant of species *b*, β_{bb} is the intraspecific competition coefficient for species *b*, and β_{ba} is the interspecific competition coefficient, or the effect of species *a* on the yield of species *b*.

These models are fit with multiple linear regression, and the coefficient estimates are used to interpret interference relationships between species. Using the Spitters method (1983), the relative competitive ability value (RC) of each species is

$$RC_a = \beta_{aa}/\beta_{ab} \text{ and } RC_b = \beta_{bb}/\beta_{ba}. \quad [3]$$

If, for example, β_{aa} is three times greater than β_{ab}, $RC_a = 3$, meaning that one plant of species *a* and three plants of species *b* have an equal influence on the average weight per plant of species *a*.

The Spitters method (1983) calculates the niche differentiation value (ND) from the RC of each species:

$$\begin{aligned} ND_{ab} &= (\beta_{aa}/\beta_{ab})/(\beta_{ba}/\beta_{bb}) = RC_a \cdot RC_b \\ &= (\beta_{aa}/\beta_{ab}) \cdot (\beta_{bb}/\beta_{ba}). \end{aligned} \quad [4]$$

Niche differentiation increases as ND departs from unity; that is, species *a* and *b* are decreasingly limited by the same resources (Spitters 1983). This interpretation assumes that both species have equal resource-utilization efficiency.

Positive interference (one-way = facilitation, two-way = mutualism) among plants is well documented (Bertness and Callaway 1994; Caldwell et al. 1998; Carpinelli et al. 2004; Choler et al. 2001; Hunter and Aarssen 1988; Mangold 2004; Pugnaire and Haase 1996; Wilson and Agnew 1992). In Equations 1 and 2, positive interference produces a negative competition coefficient. The Spitters method of quantifying relative competitive ability and niche differentiation is not appropriate where one or more competition coefficients are negative. This is because the association between a competition coefficient and its sign (+ or −) is lost in the forming of ratios. Two species exhibiting positive interference on each other produce a negative/negative ratio of competition coefficients and a positive RC. Conversely, two species exhibiting negative interference (one-way = amensalism, two-way = competition; positive competition coefficients) also produce a positive RC. That is, where both RCs are negative, a positive ND results—indistinguishable from a positive ND produced from two positive RCs.

The Spitters method is also inappropriate where one of the competition coefficients is zero or is not a statistically significant component of the regression model. Previously, where a competition coefficient was not statistically significant, it was assigned the value of zero (Jacobs et al. 1996; Mangold 2004; Roush 1988; Sheley and Larson 1994, 1996). Where the ratio of competition coefficients comprising an RC contains a zero in the numerator, the RC is zero, and the value of the denominator is moot. Where the competition coefficient in the denominator is zero, RC is undefined. In the past, researchers substituted an arbitrary, very small, competition coefficient value (i.e., 0.0001) for zero, so as to avoid an undefined RC, oftentimes producing RCs orders of magnitude greater than corresponding nonzero competition coefficients (Jacobs et al. 1996; Mangold 2004; Roush 1988; Sheley and Larson 1994, 1996).

The absolute-log method, introduced here, conserves the quantity and quality of interference while calculating values of relative competitive ability and niche differentiation and does not create moot or undefined values where one of the competition coefficients is zero. The absolute-log method also includes an optional statistical procedure constructing confidence intervals around values of relative competitive ability and niche differentiation by incorporating the variability associated with each competition coefficient estimate in subsequent calculations.

METHODS

Method Development. The first step in avoiding canceling of signs is to replace multiplication and division in RC and ND calculations with addition and subtraction, respectively, of the $\log_{10}$ of the competition coefficients. However, the $\log_{10}$ of a number ≤ 0 is undefined, and positive numbers between 0 and 1 have negative $\log_{10}$ values. To overcome this, the absolute-log function converts a competition coefficient (β) to a pseudologarithmic corollary, or $L\beta$. All positive competition coefficients have positive $L\beta$s, even those between 0 and 1, and all negative competition coefficients have negative $L\beta$s. As a result, the association between a competition coefficient and its sign is maintained during subsequent calculations of addition and subtraction.

Note that the logarithm of 1 is 0, and the logarithm of a number between 0 and 1 is negative. Also, doubling any number greater than 0 increases its logarithm by 0.301 ($\log_{10}2$). To create a scale of positive, logarithm-based corollaries for competition coefficients ≥ 1, one could simply add 1 to the competition coefficient and take the logarithm. But adding 1 before taking the logarithm distorts this relationship between increasing num-

Table 1. Competition coefficients are rescaled by dividing by the absolute value of the nonzero competition coefficient of smallest magnitude. In this case, all competition coefficients are divided by $|-0.2|$. The absolute-log function (Equations 5–7) is applied after rescaling.

Competition coefficient (β)	Rescaled coefficient (β_R)	Absolute-log (Lβ)
−0.2	−1	−1
0.5	2.5	1.398
2	10	2
0	0	0

bers and their respective logarithms. However, this relationship is maintained when taking the logarithms first, then adding 1 to them. The corresponding procedure for competition coefficients ≤ -1 requires taking the log of the coefficient's absolute value, multiplying it by -1 (to restore the negative sign) and then subtracting 1. Competition coefficients of 0 are assigned an Lβ of 0. Treatment of nonzero competition coefficients between -1 and 1 is addressed in the Rescaling section.

Rescaling. Because the logarithm of a number between 0 and 1 is negative, it is necessary to rescale competition coefficients to maintain their signs (positive or negative). Before the absolute-log function is applied, all competition coefficients (β_{aa}, β_{ab}, β_{bb}, β_{ba}) from both models (Equations 1 and 2) are divided by the absolute value of the nonzero competition coefficient closest to zero (smallest magnitude, regardless of sign) from both models. By doing so, the rescaled competition coefficients (β_R) that are nonzero have a magnitude of 1 or greater, and competition coefficients of zero remain zero (Table 1). This step avoids the discontinuity between -1 and 1 (Figure 1) and maintains proportionality among competition coefficients while conserving their respective signs.

The Absolute-Log Function. The absolute-log function, expressed algebraically, is

$$L\beta = (-1 \cdot \log|\beta_R|) - 1, \quad \text{for } \beta_R \leq -1; \qquad [5]$$

$$L\beta = 0, \quad \text{for } \beta_R = 0; \quad \text{and} \qquad [6]$$

$$L\beta = (\log \beta_R) + 1, \quad \text{for } \beta_R \geq 1. \qquad [7]$$

The RC and ND corollaries in the absolute-log method, LRC and LND, respectively, are calculated as follows:

$$LRCa = L\beta aa - L\beta ab \qquad [8]$$

$$LRCb = L\beta bb - L\beta ba \qquad [9]$$

$$LNDab = LRCa + LRCb. \qquad [10]$$

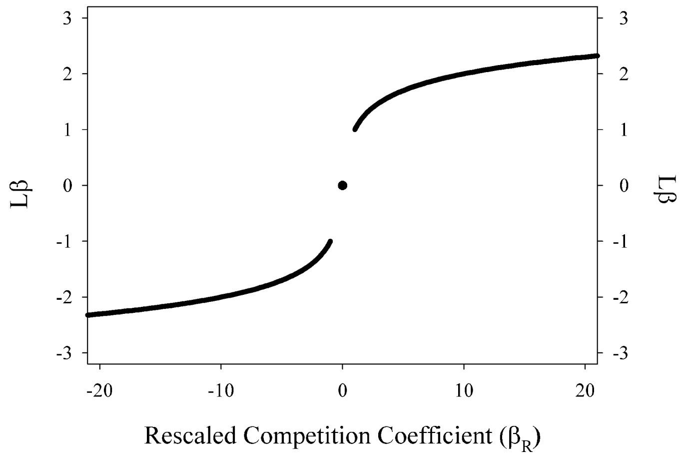

Figure 1. Plot of Lβ as a function of the rescaled competition coefficient (β_R). Where $\beta_R \leq -1$, $L\beta = (-1 \times \log|\beta_R|) - 1$; where $\beta_R = 0$, $L\beta = 0$; and where $\beta_R \geq 1$, $L\beta = (\log \beta_R) + 1$.

Interpreting Values of Relative Competitive Ability and Niche Differentiation. Where intraspecific interference equals interspecific interference, RC = 1 and LRC = 0; where intraspecific interference is greater than interspecific interference, RC > 1 and LRC > 0; and where interspecific interference is greater than intraspecific interference, RC < 1 and LRC < 0. Where there is complete niche overlap (no niche differentiation), ND = 1 and LND = 0, that is, niche differentiation increases as ND diverges from 1 and as LND diverges from 0.

A logarithmic scale is not as intuitive as a linear scale in interpreting the degree of relative competitive ability or niche differentiation. For this reason, the absolute-antilog (A-A) function is used to convert LRCs and LNDs back to nonlogarithmic, base-10-scaled corollaries, ARCs and ANDs, respectively, while conserving the sign of their respective LRCs or LNDs:

$$Ax = -1 \cdot (10^{|Lx|}), \quad \text{for } Lx \leq -1; \qquad [11]$$

$$Ax = 0, \quad \text{for } Lx = 0; \quad \text{and} \qquad [12]$$

$$Ax = 10^{Lx}, \quad \text{for } Lx \geq 1, \qquad [13]$$

where Lx = LRC or LND and Ax = ARC or AND, respectively.

Table 2. RC or ND values and their corresponding LRC or LND and ARC or AND values.[a]

Parameter	Value				
RC or ND	1/3	1/2	1	2	3
LRC or LND	−0.477	−0.301	0	0.301	0.477
ARC or AND	−3	−2	0	2	3

[a] Abbreviations: AND, absolute-antilog function corollary to ND; ARC, absolute-antilog function corollary to RC; LND, absolute-log method corollary to ND; LRC, absolute-log method corollary to RC; ND, niche differentiation value; RC, relative competitive ability value.

Table 3. Calculation of relative competitive ability and niche differentiation. β is the competition coefficient from Spitters (1983) expanded reciprocal yield law (Equations 1–4). In this example, ARCs and AND of the absolute-log method are identical to RCs and ND, respectively, of the Spitters method.[a]

Spitters		β_R	Absolute-log		A-A	
β*aa*	1.0	2	Lβ*aa*	1.301		
β*ab*	0.5	1	Lβ*ab*	1.000		
RC*a* = β*aa*/β*ab*	2.0		LRC*a* = Lβ*aa* − Lβ*ab*	0.301	ARC*a*	2.0
β*bb*	2.0	4	Lβ*bb*	1.602		
β*ba*	1.0	2	Lβ*ba*	1.301		
RC*b* = β*bb*/β*ba*	2.0		LRC*b* = Lβ*bb* − Lβ*ba*	0.301	ARC*b*	2.0
ND*ab* = RC*a* · RC*b*	4.0		LND*ab* = LRC*a* + LRC*b*	0.602	AND*ab*	4.0

[a] Abbreviations: A-A, absolute-antilog; AND, absolute-antilog function corollary to ND; ARC, absolute-antilog function corollary to RC; LND, absolute-log method corollary to ND; LRC, absolute-log method corollary to RC; ND, niche differentiation value; RC relative competitive ability value.

As with LRCs and LNDs, where intraspecific interference is greater than interspecific interference, ARC > 0; where interspecific interference is greater than intraspecific interference, ARC < 0; and where there is complete niche overlap, AND = 0 (Table 2). In the Spitters method, RCs or NDs of equal magnitude and opposite quality are reciprocals of each other. In the absolute-log method, ARCs or ANDs of equal magnitude and opposite quality have equal magnitude and opposite signs. For example, where species *a* has twice the influence on itself than species *b* has on species *a*, RC*a* = 2 and ARC*a* = 2. Where species *b* has twice the influence on species *a* than species *a* has on itself, RC*a* = 1/2 and ARC*a* = −2. Also, where two pairs of species have NDs of 3 and 1/3, they are equally niche differentiated; ANDs of these same two pairs of species are 3 and −3, respectively.

Method Comparison. Where all competition coefficients are positive and significant, the Spitters method works well and is simpler than the absolute-log method. In such a case, relative competitive ability and niche differentiation have identical values in the Spitters method (RC and ND) and the absolute-log method (ARC and AND) (Table 3).

Table 5. Calculation of relative competitive ability and niche differentiation in a case where one or more of the competition coefficients are zero. The value of β*ab* is moot as RC*a* is 0, regardless of the value of the denominator. RC*b* is undefined (*) because the denominator is 0.[a]

Spitters		β_R	Absolute-log		A-A	
β*aa*	0	2	Lβ*aa*	0		
β*ab*	3	1	Lβ*ab*	1.000		
RC*a*	0		LRC*a*	−1.000	ARC*a*	−10
β*bb*	6	4	Lβ*bb*	1.301		
β*ba*	0	2	Lβ*ba*	0		
RC*b*	*		LRC*b*	1.301	ARC*b*	20
ND*ab*	*		LND*ab*	0.301	AND*ab*	2

[a] Abbreviations: A-A, absolute-antilog; AND, absolute-antilog function corollary to ND; ARC, absolute-antilog function corollary to RC; LND, absolute-log method corollary to ND; LRC, absolute-log method corollary to RC; ND, niche differentiation value; RC relative competitive ability value.

In a comparison of two scenarios that differ only in that the strengths of interference of the competition coefficients are reversed (substituted for each other) in each model of relative competitive ability, one would intuitively expect a niche differentiation value of similar magnitude, but of opposite quality (reciprocals, Spitters; opposite sign, absolute-log). This is true in the case where all competition coefficients are positive (Table 4). Again, both methods yield equivalent results, with the Spitters method being simpler than the absolute-log method.

In the Spitters method, where the numerator of RC is 0, the value of the denominator is moot—RC is 0 regardless of the value of the denominator (Table 5). Even more problematic is a situation where a denominator is 0 and RC and ND are undefined using the Spitters method (Table 5). As in previous studies (Jacobs et al. 1996; Mangold 2004; Roush 1988; Sheley and Larson 1994, 1996), where a very small number is arbitrarily chosen to substitute for a denominator of 0, the resulting RC and ND are respectively and arbitrarily large (Table 6). Because the absolute-log method adds and subtracts pseudologarithmic corollaries, the value of the denominator is conserved and undefined values do not exist.

Table 4. Scenario 2 is identical to Scenario 1 except that the strengths of interference are reversed for both competition coefficients in both models of relative competitive ability. As a result, niche differentiation values reversed in both the Spitters method and the absolute-log method.

Scenario 1							Scenario 2						
Spitters		β_R	Absolute-log		A-A[a]		Spitters		β_R	Absolute-log		A-A	
β*aa*	1.0	2	Lβ*aa*	1.301			β*aa*	0.5	1	Lβ*aa*	1.000		
β*ab*	0.5	1	Lβ*ab*	1.000			β*ab*	1.0	2	Lβ*ab*	1.301		
RC*a*	2.0		LRC*a*	0.301	ARC*a*	2.0	RC*a*	0.5		LRC*a*	−0.301	ARC*a*	−2.0
β*bb*	2.0	4	Lβ*bb*	1.602			β*bb*	1.0	2	Lβ*bb*	1.301		
β*ba*	1.0	2	Lβ*ba*	1.301			β*ba*	2.0	4	Lβ*ba*	1.602		
RC*b*	2.0		LRC*b*	0.301	ARC*b*	2.0	RC*b*	0.5		LRC*b*	−0.301	ARC*b*	−2.0
ND*ab*	4.0		LND*ab*	0.602	AND*ab*	4.0	ND*ab*	0.25		LND*ab*	−0.602	AND*ab*	−4.0

[a] Abbreviations: A-A, absolute-antilog; AND, absolute-antilog function corollary to ND; ARC, absolute-antilog function corollary to RC; LND, absolute-log method corollary to ND; LRC, absolute-log method corollary to RC; ND, niche differentiation value; RC relative competitive ability value.

Table 6. Calculation of relative competitive ability and niche differentiation in a case where 0.0001 has been substituted for a competition coefficient of 0 in the denominator. RC and ND are dependent on the value of the substituted coeffcient. In this case, ND differs from AND by more than three orders of magnitude.[a]

Spitters		β_R	Absolute-log		A-A	
β*aa*	3.0	1.5	Lβ*aa*	1.176		
β*ab*	6.0	3.0	Lβ*ab*	1.477		
RC*a*	0.5		LRC*a*	−0.301	ARC*a*	−2
β*bb*	2.0	1.0	Lβ*bb*	1.000		
β*ba*	0 (0.0001)	0.0	Lβ*ba*	0.000		
RC*b*	20,000		LRC*b*	1.000	ARC*b*	10
ND*ab*	10,000		LND*ab*	0.699	AND*ab*	5

[a] Abbreviations: A-A, absolute-antilog; AND, absolute-antilog function corollary to ND; ARC, absolute-antilog function corollary to RC; LND, absolute-log method corollary to ND; LRC, absolute-log method corollary to RC; ND, niche differentiation value; RC relative competitive ability value.

While some of the following scenarios represent atypical situations (e.g., positive interference in all cases), they further demonstrate the universality of the absolute-log method. Consider a case where some competition coefficients are negative. In a comparison of two scenarios that differ in that the signs are reversed for both competition coefficients in one model only, one would expect the niche differentiation value to change, as is the case using the absolute-log method (Table 7). Using the Spitters method, ND remains unchanged. This is because a negative/negative RC ratio and a positive/positive RC ratio both produce a positive result. In a comparison of two scenarios that differ in that strengths of interference are reversed, as well as all signs, in each model, one would intuitively expect the niche differentiation value to remain unchanged. In such a case, AND does not change, but in the Spitters method, the signs cancel and ND changes (Table 8).

Statistical Analysis. Confidence intervals for values of relative competitive ability and niche differentiation are constructed by incorporating the variability associated with the competition coefficients from Equations 1 and 2 into subsequent calculations of ARC and AND. The distribution of each of these coefficients has a mean and standard error (SE) associated with it. Assuming these distributions are normal, the mean and SE of each coefficient is used to generate a population of coefficients from which corresponding populations of Lβs, LRCs, LNDs, ARCs, and ANDs are calculated—each with their own mean and standard deviation, from which a confidence interval (CI) can be constructed.

Constructing Confidence Intervals. For each competition coefficient from Equations 1 and 2 (β_{aa}, β_{ab}, β_{bb}, β_{ba}), create a large (e.g., 1,000) population of β values by randomly selecting from a normal distribution with mean and standard error equal to that of its respective β value. The absolute-log function (Equations 5–7) is applied to these populations of β values (after rescaling), creating corresponding populations of Lβs. From these populations of Lβs, populations of LRCs (Equations 8 and 9), LNDs (Equation 10), ARCs, and ANDs (Equations 11–13) are derived. Where these populations are normally distributed, confidence intervals are constructed as follows:

$$\mathrm{CI} = \mathrm{mean} \pm Sx, \qquad [14]$$

where S = standard error of the mean and $x = 1.645$, 1.960, or 2.576 for a 90, 95, or 99% CI, respectively. Where these populations are not normally distributed, confidence intervals are constructed using an appropriate distribution or by employing distribution-free (nonparametric) techniques.

Interpreting Confidence Intervals. Confidence intervals may be used to determine if two species differ significantly in niche (AND $\neq$ 0) or if one species is relatively more competitive than the other (ARC $\neq$ 0). For example, if the 95% CI for AND*ab* is 2.9 ± 1.7, one could say with 95% confidence that species *a* and *b* differ in niche because 0 (complete niche overlap) is not contained in the CI of their AND. Conversely, if 0 were

Table 7. By reversing the signs for both competition coefficients in only one model, one would expect the niche differentiation value to change, as is the case using the absolute-log method. Using the Spitters method, the signs cancel and ND is unchanged.[a]

Scenario 1							Scenario 2						
Spitters		β_R	Absolute-log		A-A		Spitters		β_R	Absolute-log		A-A	
β*aa*	5.0	5	Lβ*aa*	1.699			β*aa*	5.0	5	Lβ*aa*	1.699		
β*ab*	7.0	7	Lβ*ab*	1.845			β*ab*	7.0	7	Lβ*ab*	1.845		
RC*a*	0.714		LRC*a*	−0.146	ARC*a*	−1.4	RC*a*	0.714		LRC*a*	−0.146	ARC*a*	−1.4
β*bb*	−1.0	−1	Lβ*bb*	−1.000			β*bb*	1.0	1	Lβ*bb*	1.000		
β*ba*	−3.0	−3	Lβ*ba*	−1.477			β*ba*	3.0	3	Lβ*ba*	1.477		
RC*b*	0.333		LRC*b*	0.477	ARC*b*	3.0	RC*b*	0.333		LRC*b*	−0.477	ARC*b*	−3.0
ND*ab*	0.238		LND*ab*	0.331	AND*ab*	2.1	ND*ab*	0.238		LND*ab*	−0.623	AND*ab*	−4.2

[a] Abbreviations: A-A, absolute-antilog; AND, absolute-antilog function corollary to ND; ARC, absolute-antilog function corollary to RC; LND, absolute-log method corollary to ND; LRC, absolute-log method corollary to RC; ND, niche differentiation value; RC relative competitive ability value.

Table 8. After reversing the strengths and signs of both competition coefficients in both models, one would expect the ND value to remain unchanged, as is the case using the absolute-log method. Using the Spitters method, the signs cancel and ND changes.[a]

Scenario 1							Scenario 2						
Spitters		β_R	Absolute-log		A-A		Spitters		β_R	Absolute-log		A-A	
β*aa*	3.0	1.5	Lβ*aa*	1.176			β*aa*	−6.0	−3.0	Lβ*aa*	−1.477		
β*ab*	6.0	3.0	Lβ*ab*	1.477			β*ab*	−3.0	−1.5	Lβ*ab*	−1.176		
RC*a*	0.5		LRC*a*	−0.301	ARC*a*	−2.0	RC*a*	2.0		LRC*a*	−0.301	ARC*a*	−2.0
β*bb*	2.0	1.0	Lβ*bb*	1.000			β*bb*	−4.0	−2.0	Lβ*bb*	−1.301		
β*ba*	4.0	2.0	Lβ*ba*	1.301			β*ba*	−2.0	−1.0	Lβ*ba*	−1.000		
RC*b*	0.5		LRC*b*	−0.301	ARC*b*	−2.0	RC*b*	2.0		LRC*b*	−0.301	ARC*b*	−2.0
ND*ab*	0.25		LND*ab*	−0.602	AND*ab*	−4.0	ND*ab*	4.0		LND*ab*	−0.602	AND*ab*	−4.0

[a] Abbreviations: A-A, absolute-antilog; AND, absolute-antilog function corollary to ND; ARC, absolute-antilog function corollary to RC; LND, absolute-log method corollary to ND; LRC, absolute-log method corollary to RC; ND, niche differentiation value; RC relative competitive ability value.

contained in the 95% CI (e.g., 2.9 ± 3.7), one could not say with 95% confidence that the two species differ in niche. In a comparison of the relative competitive abilities of species *a* and *b* on species *a*, if the 95% CI of ARC*a* = −1.4 ± 0.8, we could say with 95% confidence that the interspecific interference of species *b* on species *a* is greater than the intraspecific interference of species *a* on itself (i.e., $\beta_{ab} > \beta_{aa}$) because 0 is not contained in the CI of ARC*a.* Conversely, if the 95% CI of ARC*a* = −1.4 ± 1.6, one could not say with 95% confidence that the interspecific interference of species *b* on species *a* is greater than the intraspecific interference of species *a* on itself because 0 is contained in the CI of ARC*a.*

Spreadsheet Application. A spreadsheet file is available (author) which generates statistical analysis and graphical output for the absolute-log and Spitters methods. One worksheet simultaneously calculates RCs and NDs (Spitters method) and Lβs, LRCs, LNDs, ARCs, and ANDs (absolute-log method), allowing the user to contrast and compare different outcomes within and between the two methods for various combinations of β values. As the user creates competition coefficient populations using the random number generator, a second worksheet automatically rescales β_{aa}, β_{ab}, β_{bb}, and β_{ba}, applies the absolute-log function (creating $L\beta_{aa}$, $L\beta_{ab}$, $L\beta_{bb}$, and $L\beta_{ba}$, respectively), and calculates LRCs, LNDs, ARCs, and ANDs, including 90%, 95%, and 99% confidence intervals for each. A third worksheet calculates RCs and NDs with confidence intervals; however, because of problems associated with sign canceling and undefined values, this should only be used when all values produced by the random number generator are > 0 (a value ≤ 0 is usually produced for any $\beta > 0$ if the SE is approximately 0.3 times the mean or larger). The worksheet alerts the user if any value created by the random number generator is ≤ 0. A fourth worksheet contains case studies with interpretation.

CONCLUSION

The elegance and simplicity of the Spitters method make it the preferred approach to quantifying and interpreting relative competitive ability and niche differentiation where all competition coefficients are positive and statistically significant. In cases where a competition coefficient is negative, zero, or not statistically significant, the absolute-log method should be used. For either method, the statistical analysis procedure outlined here allows for a level of confidence to be assigned to values of relative competitive ability and niche differentiation.

Understanding interference relationships of plant species is essential in both theoretical and applied ecology. Quantifying the relative competitive ability between species under a variety of environmental conditions may provide insight into the role of plant–plant interactions in community dynamics. For example, a shift in the competitive ability of one species vs. another resulting from environmental change (e.g., climatic, hydrologic, edaphic, etc.) may contribute to the crossing of a threshold between relatively stable community states. This may be especially true where keystone species and/or invasive species are involved. Revegetation specialists often strive to preempt resources from invasive species through niche occupation and resource capture by desirable species. Knowledge of the relative competitive abilities and niche differentiation among species, both desirable and undesirable, is key to designing a diverse, competitive, revegetation seeding mixture.

ACKNOWLEDGMENTS

I thank Ed Luschei, Bruce Maxwell, Steve Radosevich, and Tony Svejcar, whose helpful comments improved this manuscript.

LITERATURE CITED

Bertness, M. D. and R. Callaway. 1994. Positive interaction in communities. Trends Ecol. Evol. 9:191–193.

Caldwell, M. M., T. E. Dawson, and J. H. Richards. 1998. Hydraulic lift: consequences of water efflux from the roots of plants. Oecologia 113: 151–161.

Carpinelli, M. F., R. L. Sheley, and B. D. Maxwell. 2004. Revegetating weed-infested rangeland with niche-differentiated desirable species. J. Range Manage. 57:97–105.

Choler, P., R. Michalet, and R. M. Callaway. 2001. Facilitation and competition on gradients in alpine plant communities. Ecology 82:3295–3308.

Hunter, A. F. and L. W. Aarssen. 1988. Plants helping plants. Bioscience 38: 34–40.

Jacobs, J. S., R. L. Sheley, and B. D. Maxwell. 1996. Effect of *Sclerotinia sclerotiorum* on the interference between bluebunch wheatgrass (*Agropyron spicatum*) and spotted knapweed (*Centaurea maculosa*). Weed Techol. 10:13–21.

Mangold, J. M. 2004. Investigating the potential of using R* theory to manage nonindigenous plant invasions. Ph.D. thesis. Montana State University, Bozeman, MT. 136 p.

Pugnaire, F. I. and P. Haase. 1996. Facilitation between higher plant species in a semiarid environment. Ecology 77:1420–1426.

Roush, M. L. 1988. Models of a four-species annual weed community: growth, competition, and community dynamics. Ph.D. thesis. Oregon State University, Corvallis, OR. 218 p.

Sheley, R. L. and L. L. Larson. 1994. Comparative growth and interference between cheatgrass and yellow starthistle seedlings. J. Range Manage. 47:470–474.

Sheley, R. L. and L. L. Larson. 1996. Emergence date effects on resource partitioning between diffuse knapweed seedlings. J. Range Manage. 49: 241–44.

Spitters, C.J.T. 1983. An alternative approach to the analysis of mixed cropping experiments. I. Estimation of competition effects. Neth. J. Agric. Sci. 31:1–11.

Wilson, J. B. and A.D.Q. Agnew. 1992. Positive switches in plant communities. Advances Ecol. Res. 23:263–336.